

Sushil Kumar Banerji
Memorial Lecture

Indian Statistical Institute
Kolkata
Feb 28, 2020

Gadadhar Misra
Indian Institute of Science
Bangalore

Act I

Looking Back

William Gladstone on seeing
Michael Faraday's experiments:

William Gladstone on seeing
Michael Faraday's experiments:

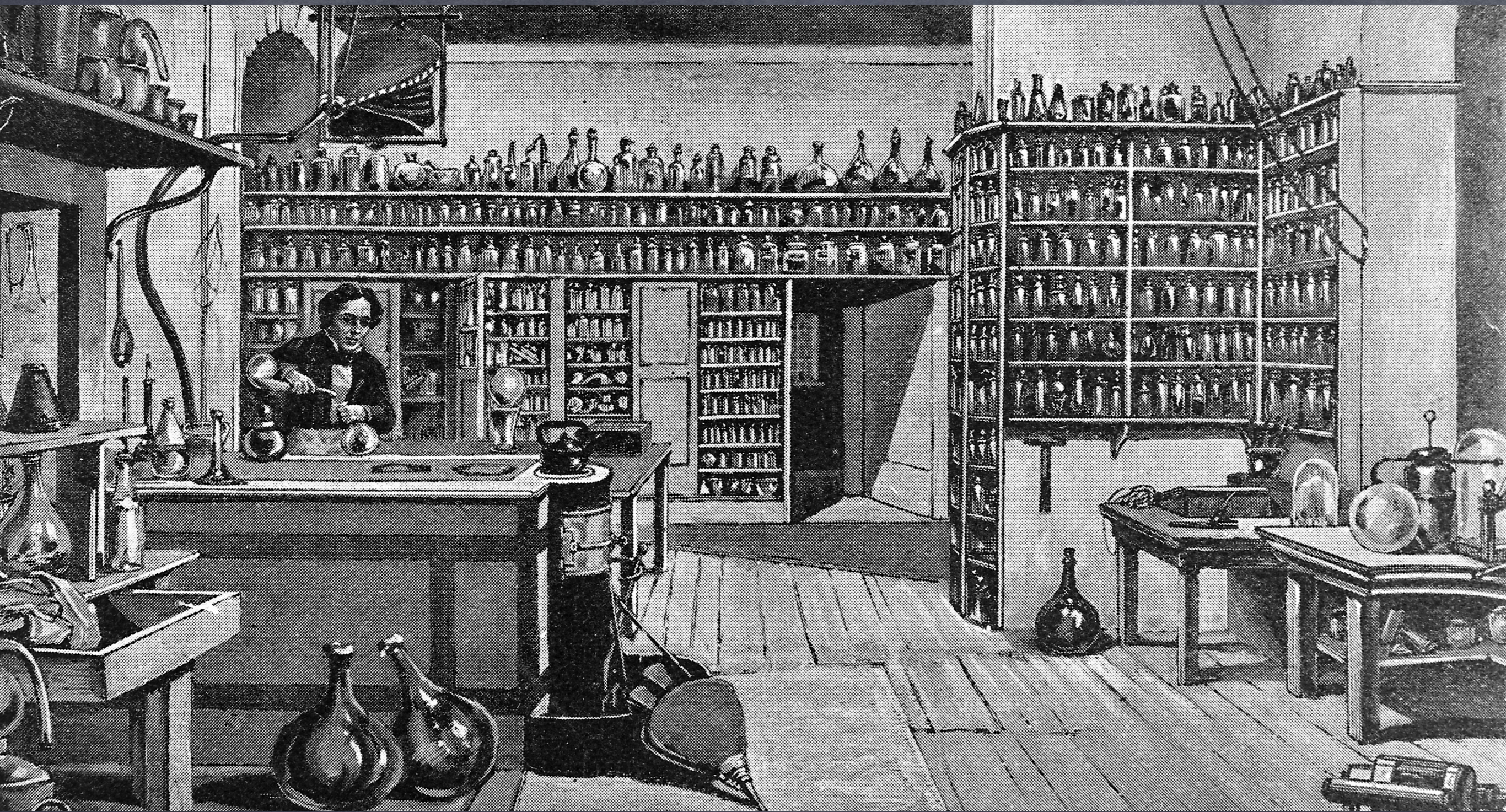
"But, after all, what use is it?"

William Gladstone on seeing
Michael Faraday's experiments:

"But, after all, what use is it?"

"Why sir, there is every probability
that you will soon be able to tax it."

Faraday in his laboratory at the Royal Institution, London.



This file comes from Wellcome Images, a website operated by Wellcome Trust, a global charitable foundation based in the United Kingdom.

Radium was discovered
by Marie Skłodowska-
Curie and her husband
Pierre Curie on 21
December 1898, in a
uraninite (pitchblende)
sample.





We must not forget that when radium was discovered no one knew that it would prove useful in hospitals.

The work was one of pure science.

Marie Curie

Lecture at Vassar College, Poughkeepsie, New York (14 May 1921). In Cambridge Editorial Partnership, *Speeches that Changed the World*, 53.

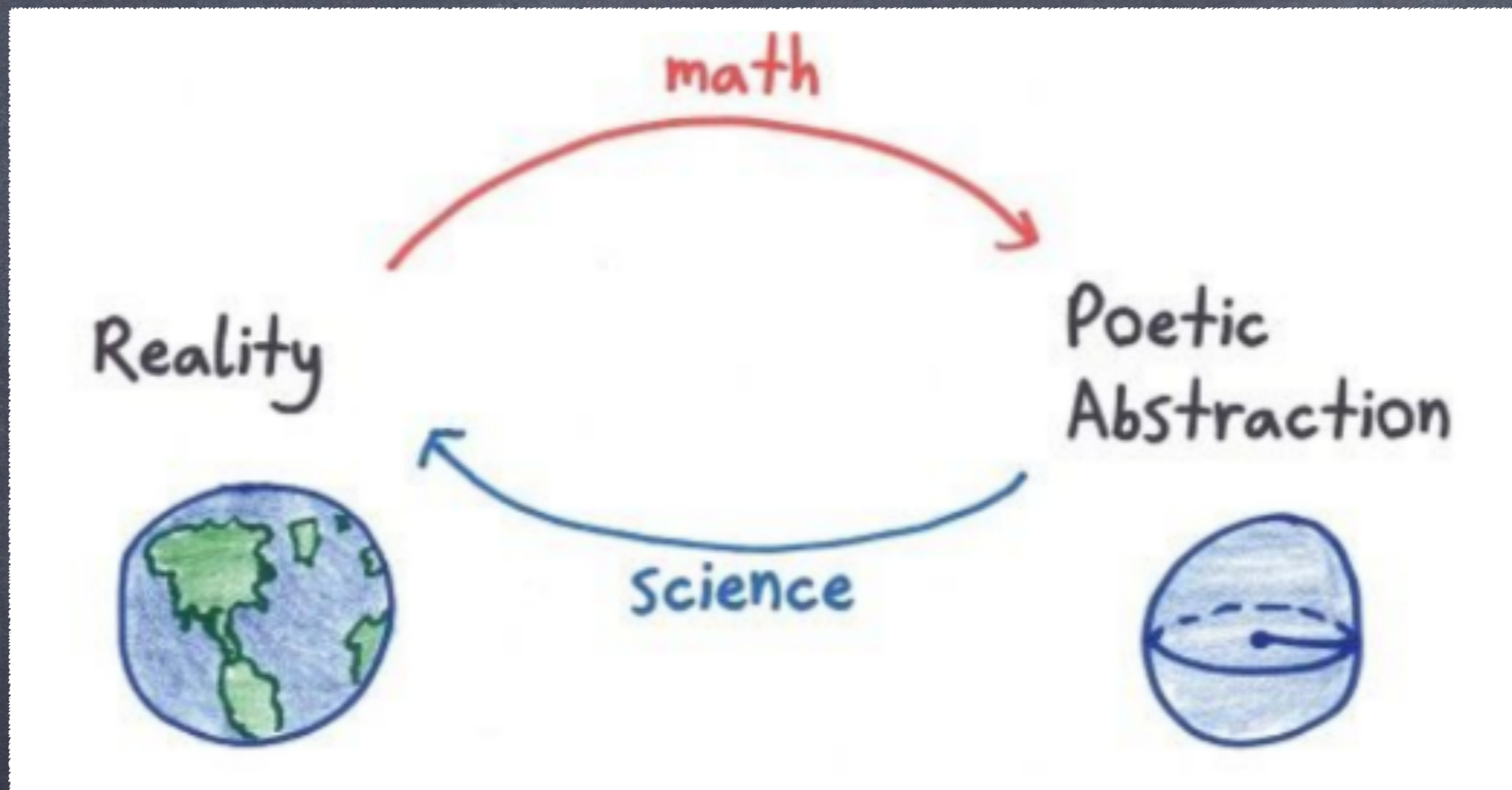
And this is a proof that scientific work must not be considered from the point of view of the direct usefulness of it.

It must be done for itself, for the beauty of science, and then there is always the chance that a scientific discovery may become like the radium a benefit for humanity.

Marie Curie

Lecture at Vassar College, Poughkeepsie, New York (14 May 1921). In Cambridge Editorial Partnership, *Speeches that Changed the World*, 53.

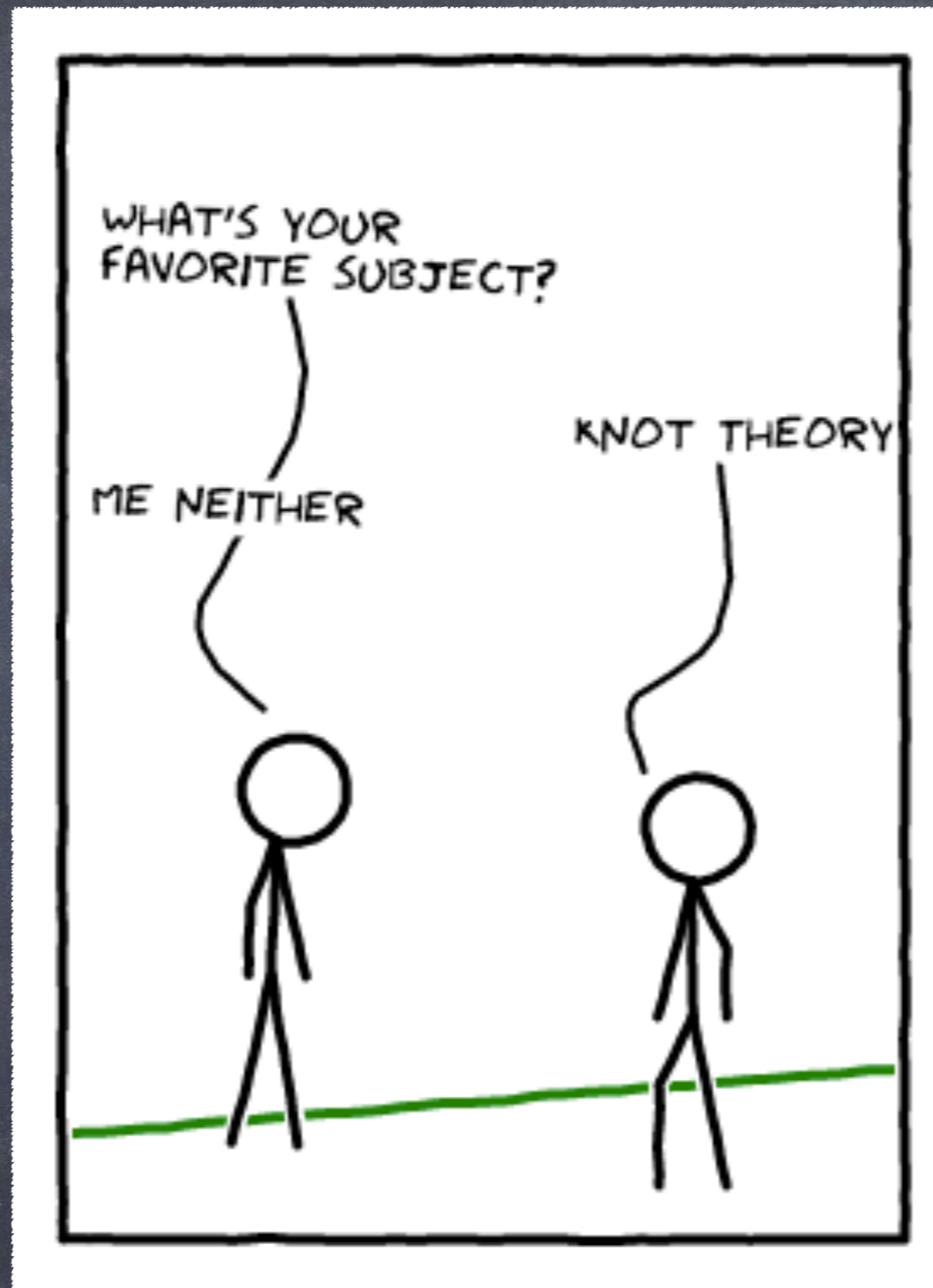
Science is about discovery not just about solving human problems.



Ben Orlin

Math with Bad Drawings:

Illuminating the Ideas that Shape Our Reality



1994, Colin C. Adams.

The Knot Book: Knot Jokes and Pastimes (attributed to Martin Scharlemann)

The branch of knot theory was originally inspired by a scientific problem - the quest to understand "ether" - before the theory of atoms came along.

The branch of knot theory was originally inspired by a scientific problem - the quest to understand "ether" - before the theory of atoms came along.

Although the questions related to knots were obsolete as far as the scientific motivation was concerned, mathematicians remained fascinated.

Robert J. Aumann did his
doctoral thesis in knot
theory, in the fifties.

"One of the things I proved in my doctoral thesis is that alternating knots do not come apart, ever. This result is very easy to state, but it was very difficult to prove.

I worked very hard on this problem, and the result had not been known before; and, it is absolutely useless.

Who cares! Right? No!! Wrong!!"

Aumann RJ. Pure science and applied science.

Rambam Maimonides Med J. 2011;2(1):e0017. Published 2011 Jan 31.

doi:10.5041/RMMJ.10017

"Grandpa, can I pick your brain?"

(October 2004.)

"Sure, Yacov. What's up?"

"What are linking numbers?"

"Linking numbers, in knot theory?"

"Yes, in knot theory."

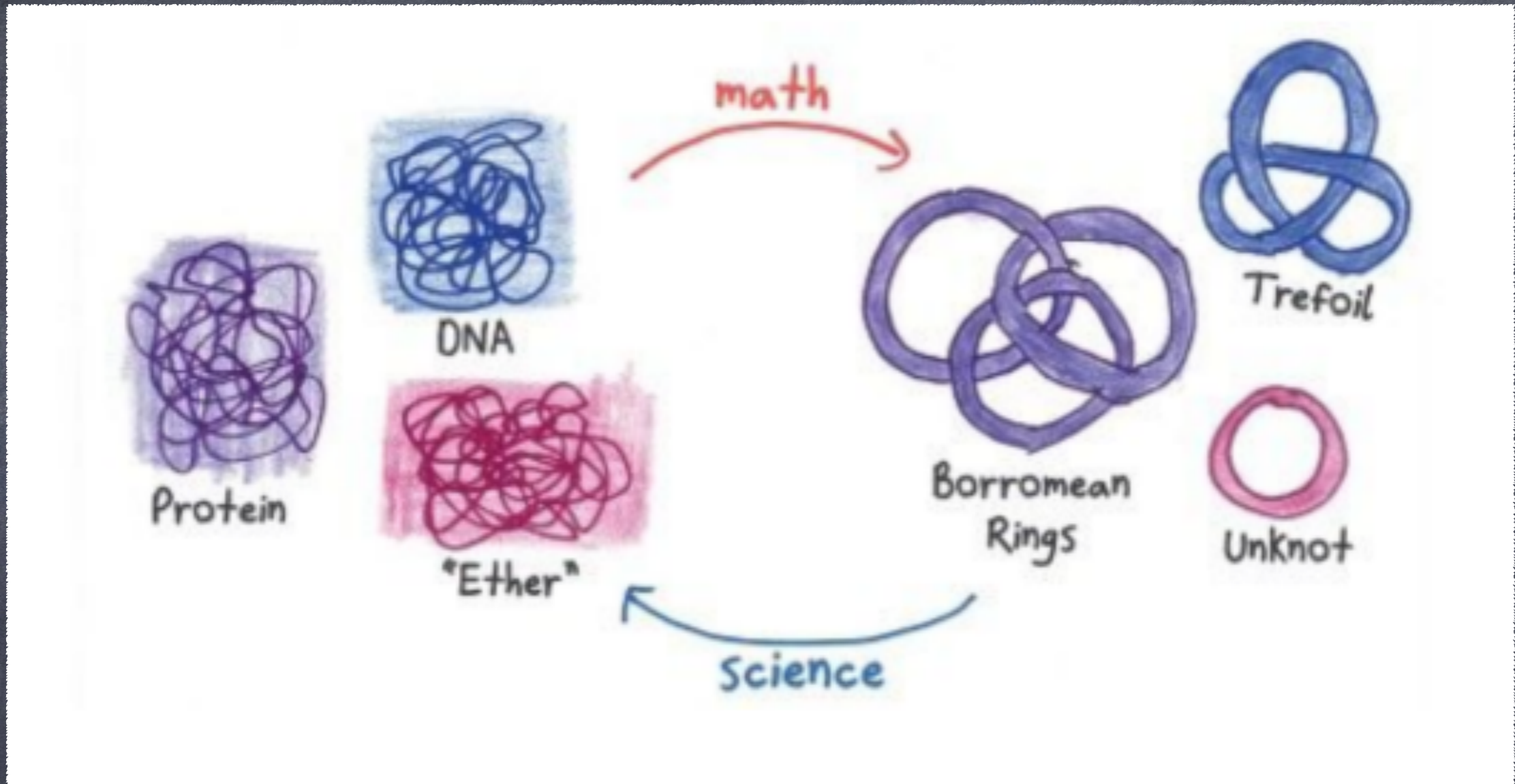
"Why are you interested in linking numbers, why are you interested in knot theory?"

"Well, we are studying knot theory in medical school, and the professor talked to us about linking numbers. I didn't understand what he was saying, and I don't think he understood it either. So I am asking you: what are linking numbers?"

Aumann RJ. Pure science and applied science.

Rambam Maimonides Med J. 2011;2(1):e0017. Published 2011 Jan 31.

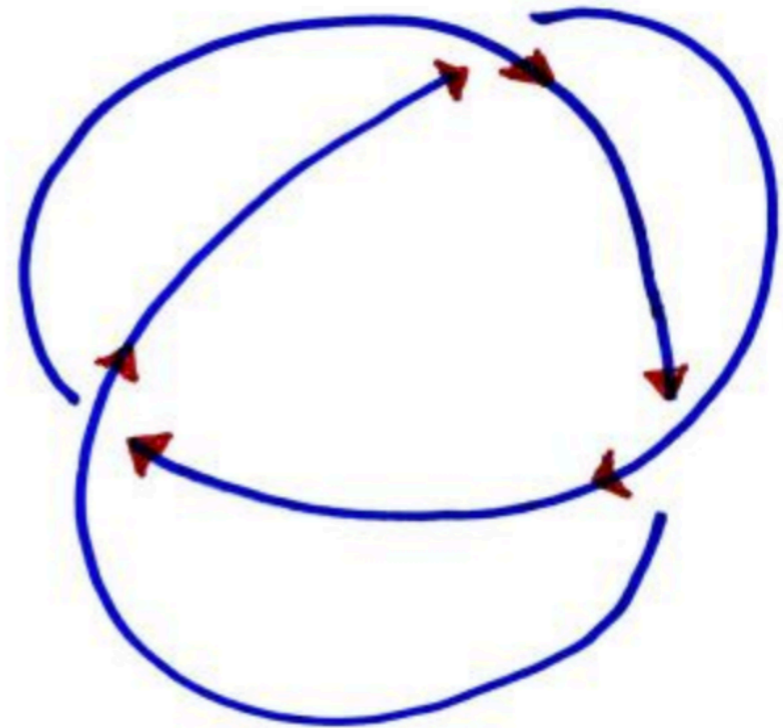
doi:10.5041/RMMJ.10017



Ben Orlin
Math with Bad Drawings:
Illuminating the Ideas that Shape Our Reality

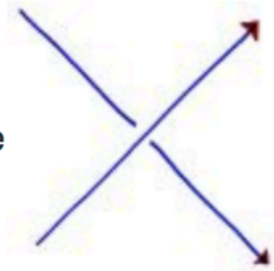
The writhe of the diagram is now the sum of all the incidence numbers.

The linking number is a link invariant!

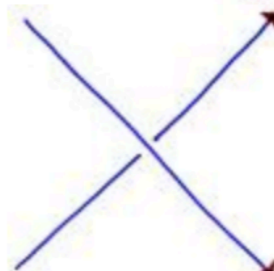


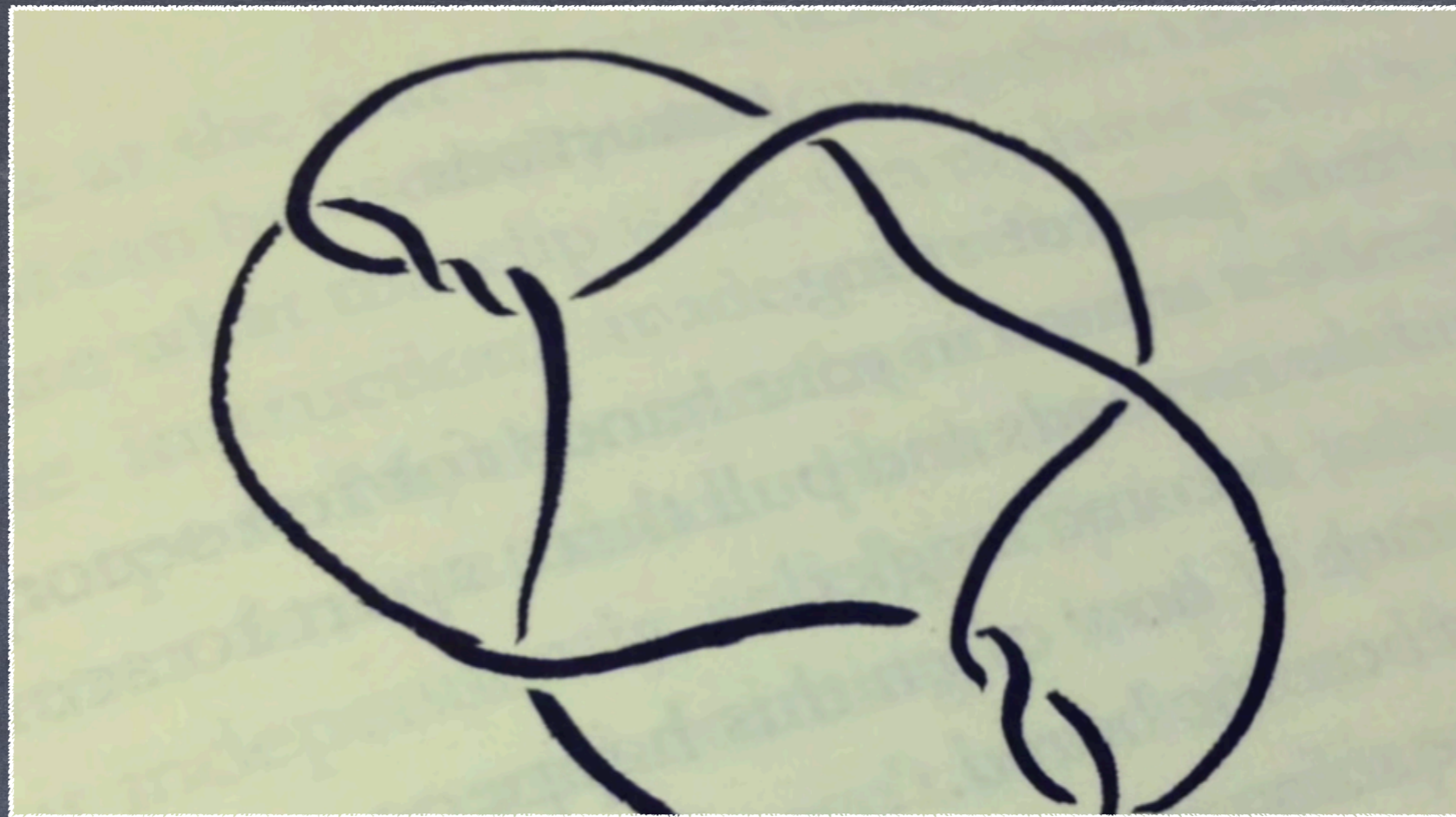
and define the *incidence number* of a crossing to be:

+1 if the crossing is like



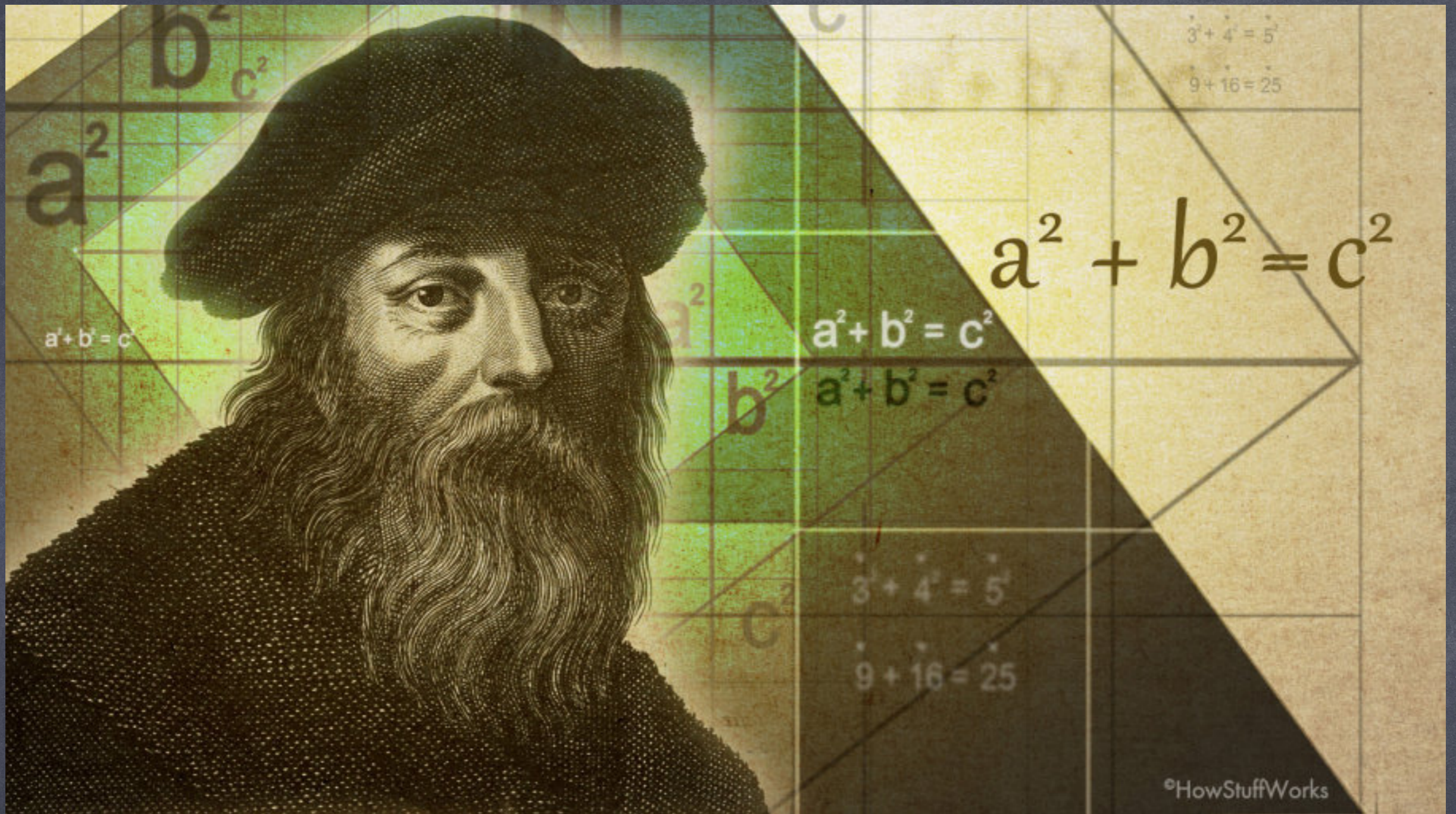
-1 if the crossing is like





At the moment, the bacteria appear to be better at undoing knots than humans.

The bacteria can snip the DNA on one side and join it back together on the other side producing a very knotted DNA.



Patrick J. Kiger

"The Pythagorean Theorem Makes Construction and GPS Possible"

8 November 2019. HowStuffWorks.com.

"This theorem and those related to it have given us our entire system of measurement: It allows pilots to navigate in windy skies, and ships to set their course. ALL GPS measurements are possible because of this theorem."

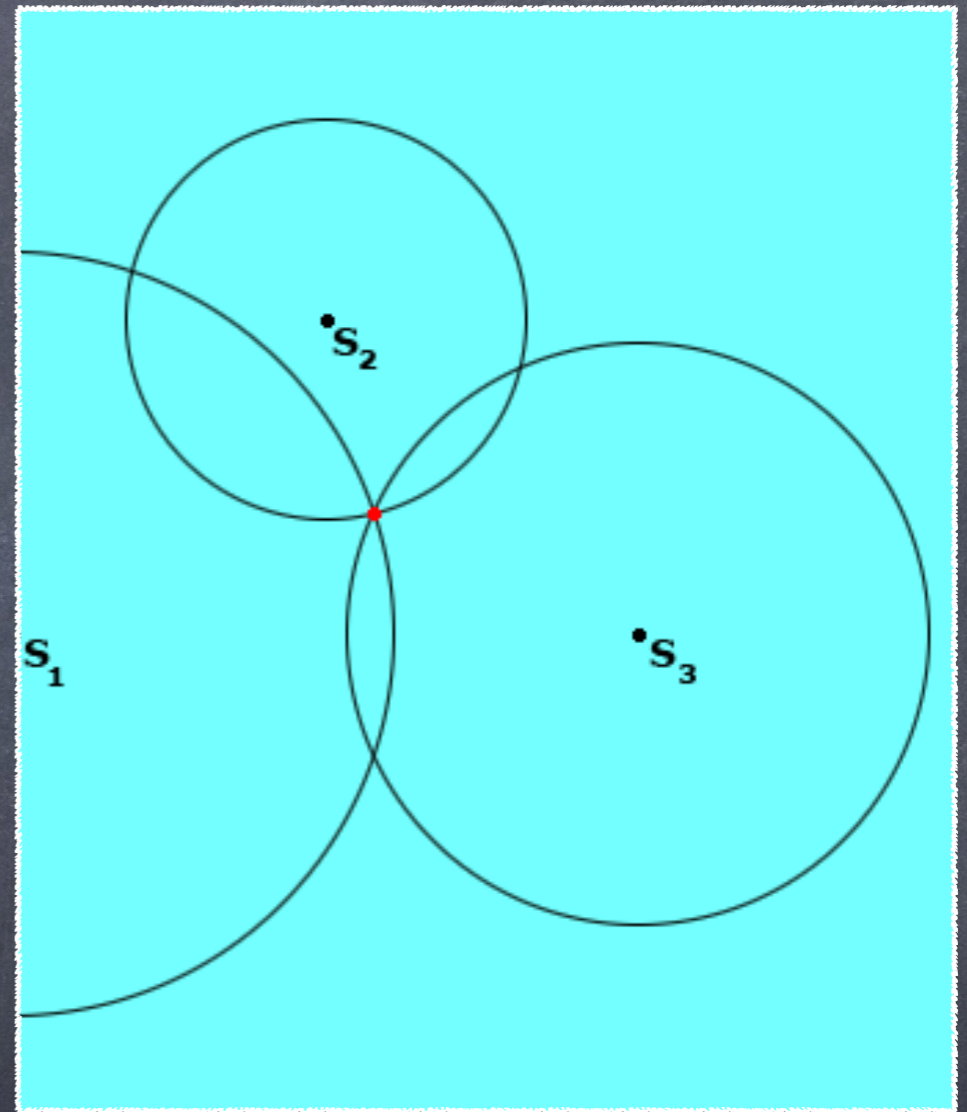
Patrick J. Kiger

"The Pythagorean Theorem Makes Construction and GPS Possible"

8 November 2019. HowStuffWorks.com.

How does the GPS locate you?

If you happen to know that you are at a distance x_1 from some place s_1 , distance x_2 from some other place s_2 and finally, at a distance x_3 from a place s_3 , then you can find your position.

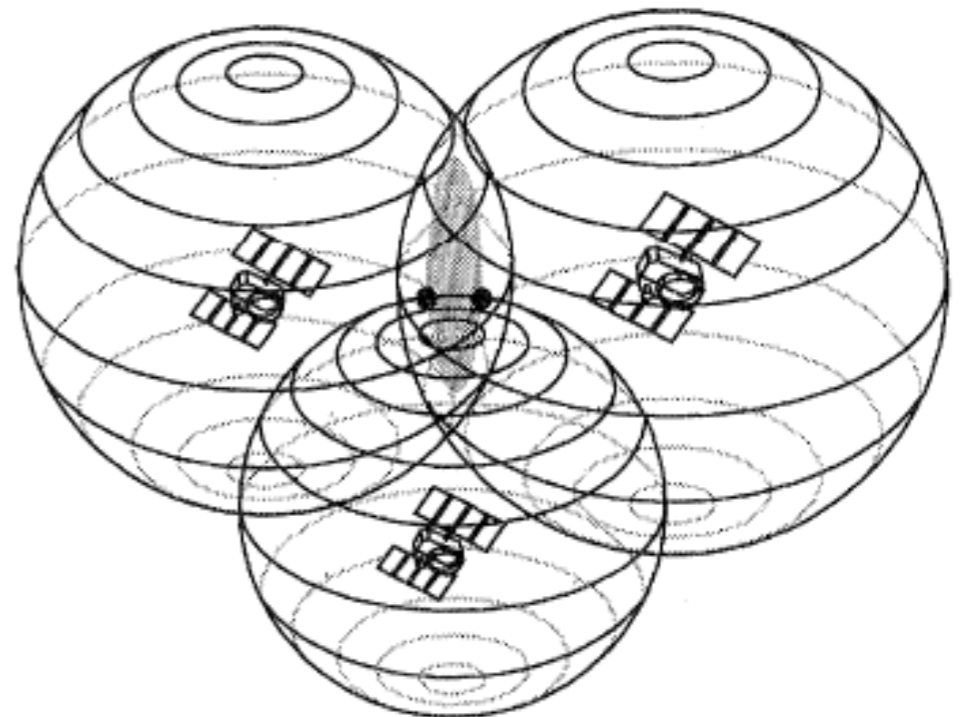
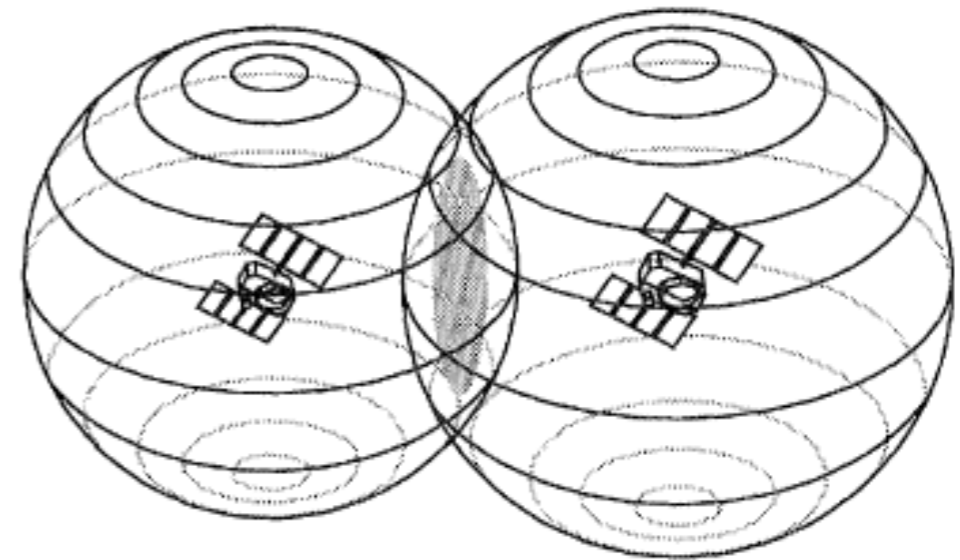


Spherical Geometry and a little bit of relativity!

Once you know three
accurate distances from
three specific locations, it is
an easy matter to plot their
intersection on a two
dimensional map.

The mathematics of GPS

- The intersection of two spheres is either empty or a circle.
- The circle will intersect a third sphere in at most two points.
- This geometric fact is the basis of GPS since other factors can be used to eliminate one of the two points as being an irrelevant solution to the problem.



Alexander Grothendieck

Clip slide



$$\begin{array}{ccc} H_n(X, A) & \xrightarrow{T} & h_n(X, A) \\ \downarrow \partial & & \downarrow \partial \\ H_{n-1}(A) & \xrightarrow{T} & h_{n-1}(A) \end{array}$$

The Grothendieck
inequality

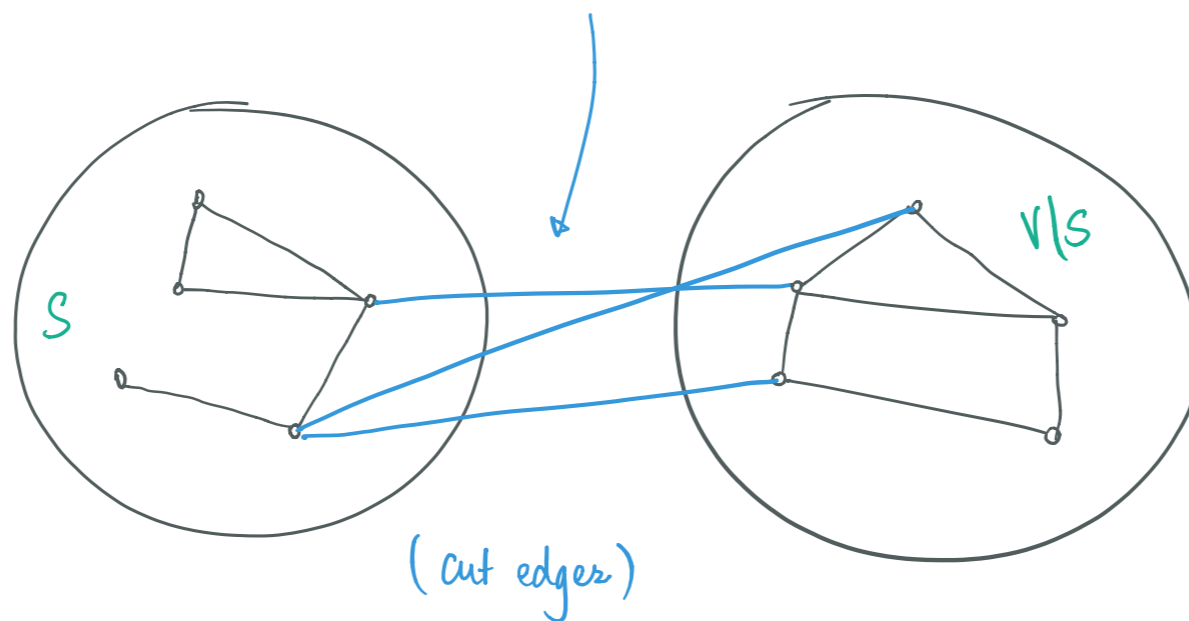
The Max-Cut
problem and
applications

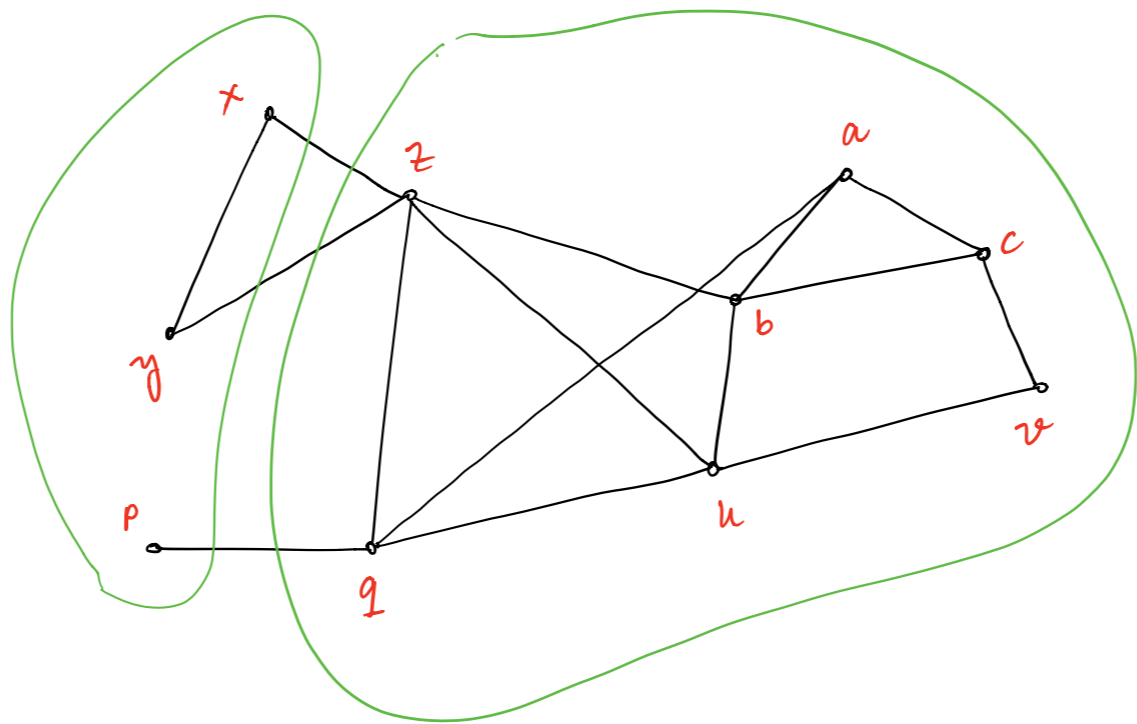
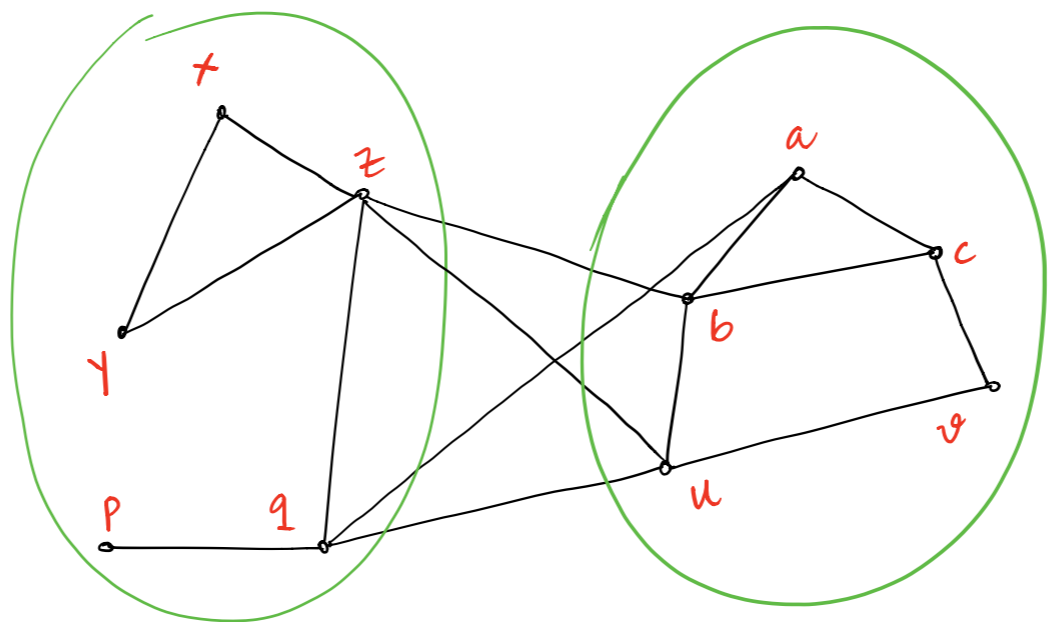
MAX CUT.

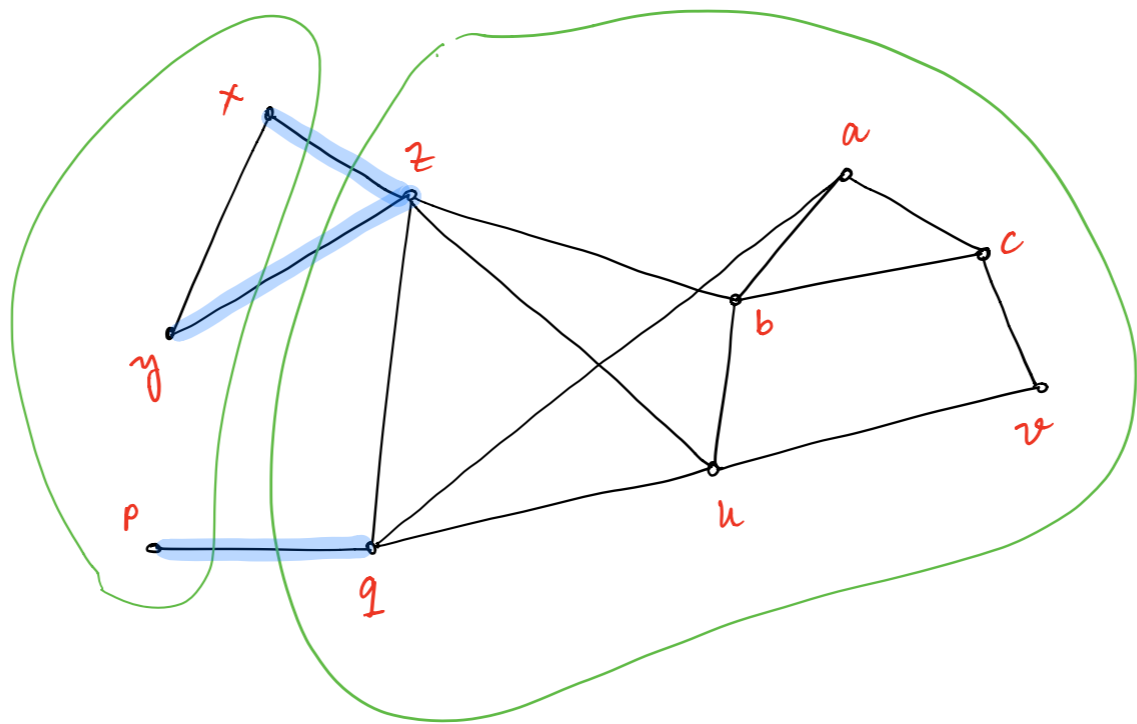
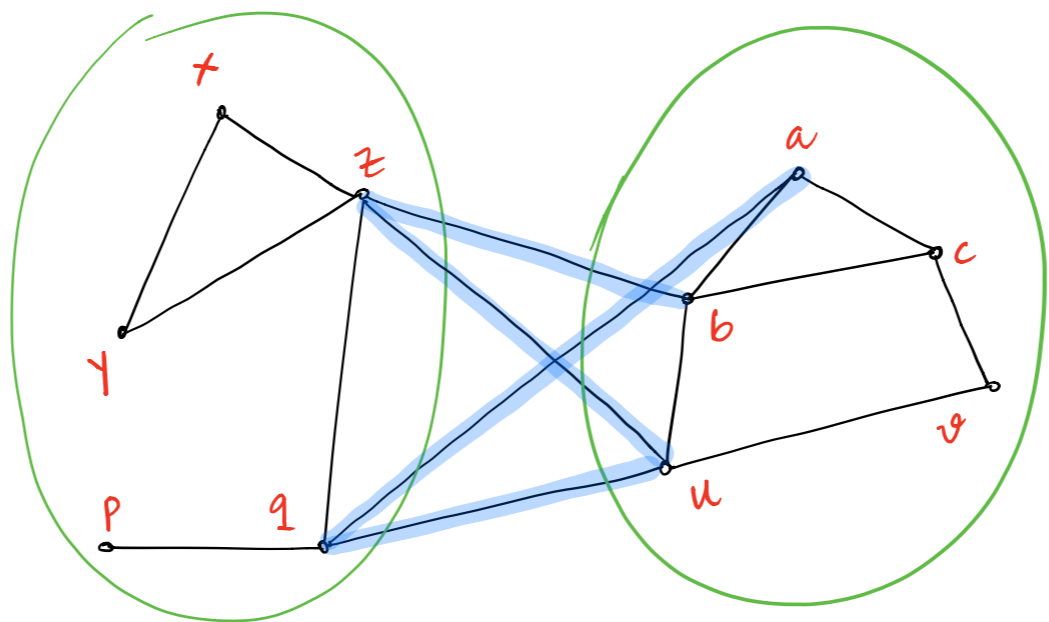
A cut in a graph $G = (V, E)$ is a pair $(S, V \setminus S)$.

The edge set of the cut is the set of all edges

$$E(S, V \setminus S) = \{e \in E \mid |e \cap S| = |e \cap V \setminus S| = 1\}$$

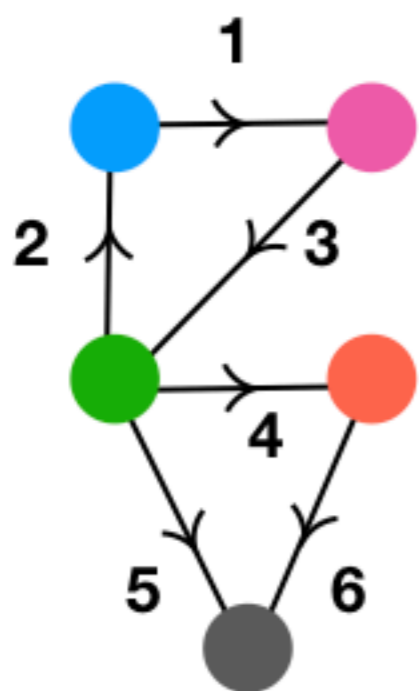


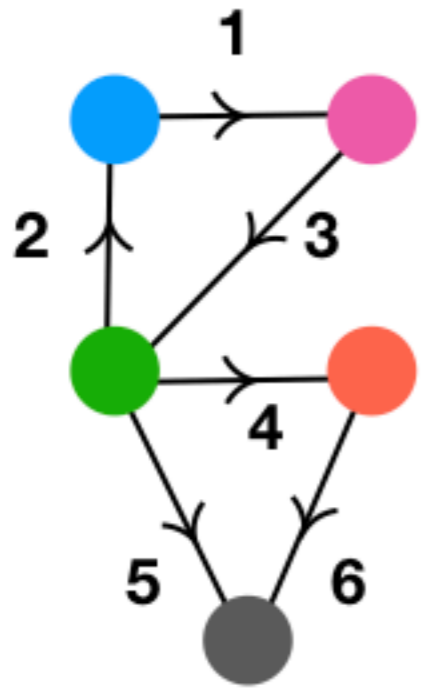


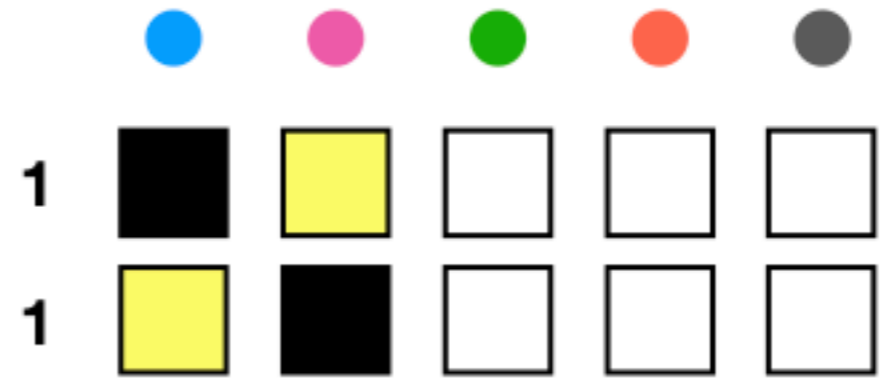
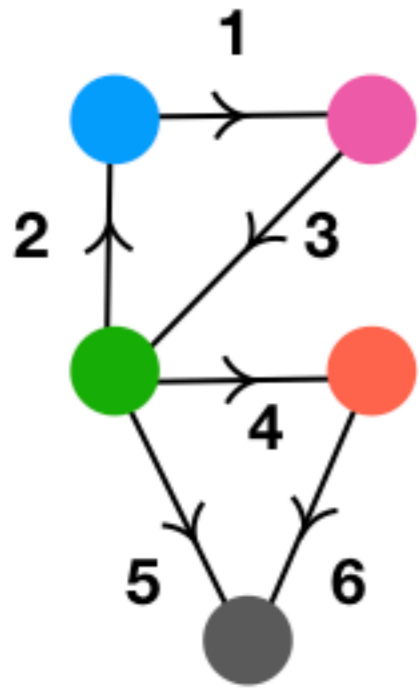


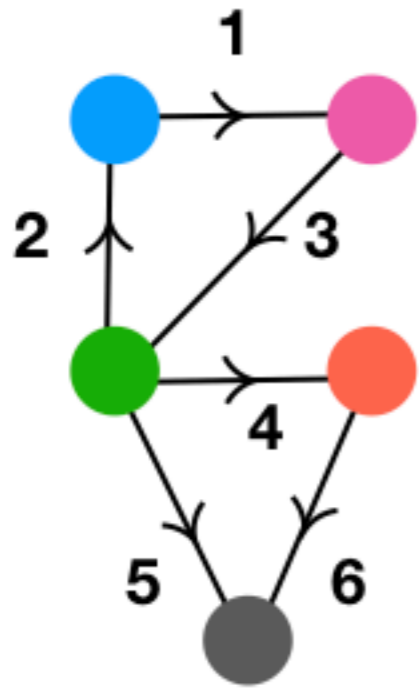
Applications of the max-cut problem


























Partitioning a set of data points into groups of closely related observations can be modelled as a max-cut problem. The points in a group or cluster should be more close or similar to each other than they are to points in the other cluster. It also occurs in the analysis of spin glass model.

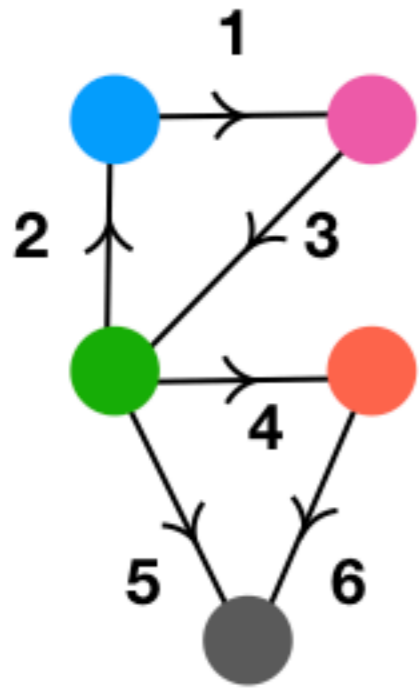







































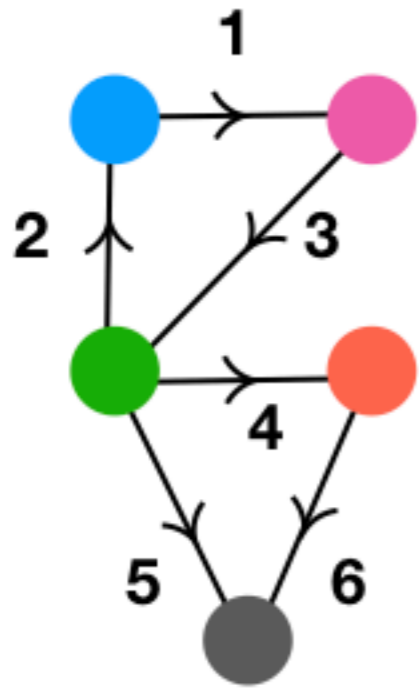




































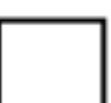












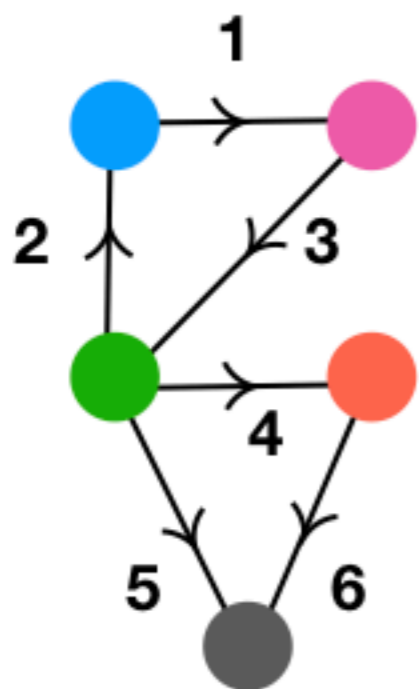
| |  |  |  |  |  |
|---|---|---|---|---|---|
| 1 |  |  |  |  |  |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 2 |  |  |  |  |  |




































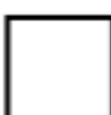





















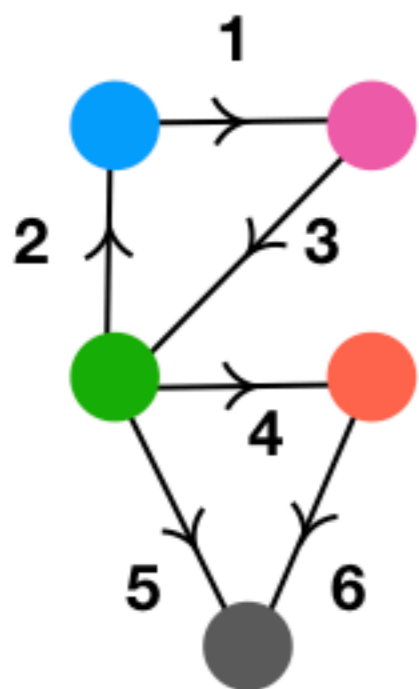
| |  |  |  |  |  |
|---|--|--|--|--|--|
| 1 |  |  |  |  |  |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 3 |  |  |  |  |  |








| |  |  |  |  |  |
|---|---|---|---|---|---|
| 1 |  |  |  |  |  |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 4 |  |  |  |  |  |



| |  |  |  |  |  |
|---|---|---|---|---|---|
| 1 |  |  |  |  |  |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 5 |  |  |  |  |  |
| 5 |  |  |  |  |  |

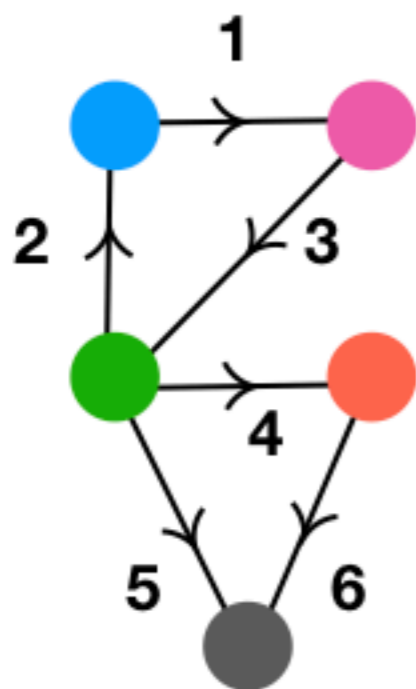





































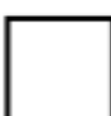



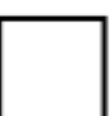















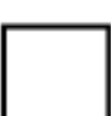
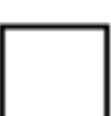








| |  |  |  |  |  |
|---|---|---|---|---|---|
| 1 | ■ | ■ | □ | □ | □ |
| 1 | ■ | ■ | □ | □ | □ |
| 2 | ■ | □ | ■ | □ | □ |
| 2 | ■ | □ | ■ | □ | □ |
| 3 | □ | ■ | ■ | □ | □ |
| 3 | □ | ■ | ■ | □ | □ |
| 4 | □ | □ | ■ | ■ | □ |
| 4 | □ | □ | ■ | ■ | □ |
| 5 | □ | □ | ■ | □ | ■ |
| 5 | □ | □ | ■ | □ | ■ |
| 6 | □ | □ | □ | ■ | ■ |
| 6 | □ | □ | □ | ■ | ■ |

Cut Norm

maximum, over all $I \subseteq R, J \subseteq S$,

$$\left| \sum_{i \in I, j \in J} a_{ij} \right|$$

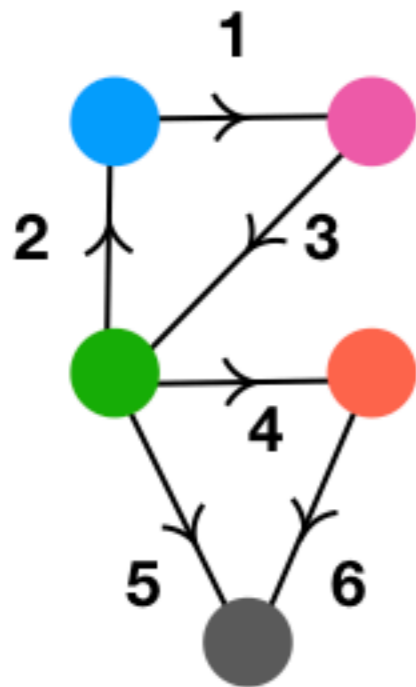


| |  |  |  |  |  |
|---|---|---|---|---|---|
| 1 |  |  |  |  |  |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 5 |  |  |  |  |  |
| 5 |  |  |  |  |  |
| 6 |  |  |  |  |  |
| 6 |  |  |  |  |  |




































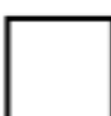



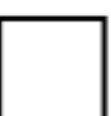















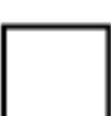
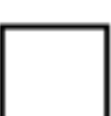



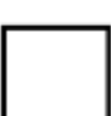
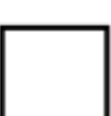
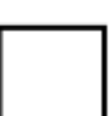


Cut Norm

maximum, over all $I \subseteq R, J \subseteq S$,

$$\left| \sum_{i \in I, j \in J} a_{ij} \right|$$



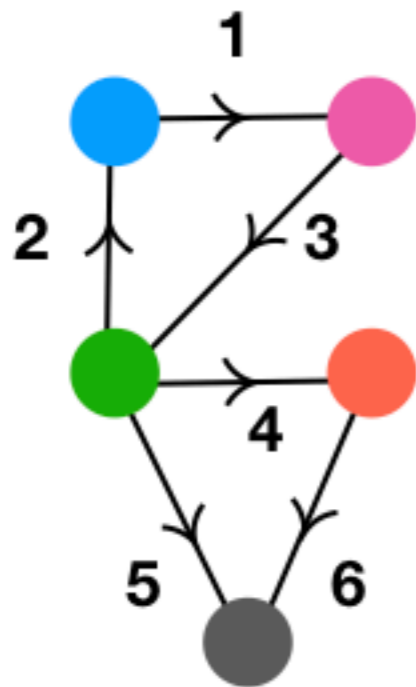
Claim: The cut norm
(of the matrix on the right)
is equal
to the size of the max cut
(of the graph on the left).

| |  |  |  |  |  |
|---|---|---|---|---|---|
| 1 |  |  |  |  |  |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 5 |  |  |  |  |  |
| 5 |  |  |  |  |  |
| 6 |  |  |  |  |  |
| 6 |  |  |  |  |  |

Cut Norm

maximum, over all $I \subseteq R, J \subseteq S$,

$$\left| \sum_{i \in I, j \in J} a_{ij} \right|$$



Claim: The cut norm
(of the matrix on the right)
is equal
to the size of the max cut
(of the graph on the left).

| 1 | | | | | |
|---|--|--|--|--|--|
| 1 | | | | | |
| 2 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 6 | | | | | |

The cut norm of a $m \times n$ matrix A is defined by setting

$$\left\| \left(a_{ij} \right) \right\|_{\text{cut}} := \max_{I, J} \left| \sum_{i \in I, j \in J} a_{ij} \right|,$$

Where $A = a_{ij}$ and the maximum is taken over all subsets I and J of the index sets: $\{1, 2, \dots, m\}$ and $\{1, 2, \dots, n\}$, respectively.

The cut-norm is the same as the size of the max-cut!

- It is not hard to show that the cut-norm is equivalent to the operator norm:

- It is not hard to show that the cut-norm is equivalent to the operator norm:
- $4\|A\|_{\text{cut}} \geq \|A\|_{\infty \rightarrow 1} \geq \|A\|_{\text{cut}}$, where

• It is not hard to show that the cut-norm is equivalent to the operator norm:

• $4\|A\|_{\text{cut}} \geq \|A\|_{\infty \rightarrow 1} \geq \|A\|_{\text{cut}}$, where

• $\|A\|_{\infty \rightarrow 1} = \sup \left\{ \left| \sum_{i,j} a_{ij} s_i t_j \right| : |s_i|, |t_j| = 1 \right\}$

- It is not hard to show that the cut-norm is equivalent to the operator norm:
- $4\|A\|_{\text{cut}} \geq \|A\|_{\infty \rightarrow 1} \geq \|A\|_{\text{cut}}$, where
- $\|A\|_{\infty \rightarrow 1} = \sup \left\{ \left| \sum_{i,j} a_{ij} s_i t_j \right| : |s_i|, |t_j| = 1 \right\}$
- Thus one may wish to compute $\|A\|_{\infty \rightarrow 1}$. But this is not easy either.

- The 2-summing norm $\gamma(A)$ of the matrix A is defined by replacing the scalars s and t in the definition of $\|A\|_{\infty \rightarrow 1}$ by unit vectors in an arbitrary Hilbert space as follows.

• The 2-summing norm $\gamma(A)$ of the matrix A is defined by replacing the scalars s and t in the definition of $\|A\|_{\infty \rightarrow 1}$ by unit vectors in an arbitrary Hilbert space as follows.

$$\gamma(A) = \sup \left\{ \left| \sum_{i,j} a_{ij} \langle x_i, y_j \rangle \right|, x_i, y_j \in (\ell^2)_1 \right\}$$

• The 2-summing norm $\gamma(A)$ of the matrix A is defined by replacing the scalars s and t in the definition of $\|A\|_{\infty \rightarrow 1}$ by unit vectors in an arbitrary Hilbert space as follows.

•
$$\gamma(A) = \sup \left\{ \left| \sum_{i,j} a_{ij} \langle x_i, y_j \rangle \right|, x_i, y_j \in (\ell^2)_1 \right\}$$

• $\gamma(A)$ is equivalent to the norm $\|A\|_{\infty \rightarrow 1}$

• The 2-summing norm $\gamma(A)$ of the matrix A is defined by replacing the scalars s and t in the definition of $\|A\|_{\infty \rightarrow 1}$ by unit vectors in an arbitrary Hilbert space as follows.

$$\gamma(A) = \sup \left\{ \left| \sum_{i,j} a_{ij} \langle x_i, y_j \rangle \right|, x_i, y_j \in (\ell^2)_1 \right\}$$

• $\gamma(A)$ is equivalent to the norm $\|A\|_{\infty \rightarrow 1}$

• Indeed taking the supremum $\frac{\gamma(A)}{\|A\|_{\infty \rightarrow 1}}$ over all $n \times n$ matrices A , we obtain the Grothendieck constant $K_G(n)$.

Act II

Current Affairs

Applied vs. Pure

Applied vs. Pure

- © 2010 International Review of
mathematical Sciences:

Applied vs. Pure

- 2010 International Review of mathematical Sciences:
- Distinction between (i) pure and applied; (ii) mathematics and statistics.

Applied vs. Pure

- 2010 International Review of mathematical Sciences:
- Distinction between (i) pure and applied; (ii) mathematics and statistics.
- They have produced an increasingly negative effect ... by stressing divisions rather than the unifying effects.

Applied vs. Pure

- 2010 International Review of mathematical Sciences:
- Distinction between (i) pure and applied; (ii) mathematics and statistics.
- They have produced an increasingly negative effect ... by stressing divisions rather than the unifying effects.
- The features that unite mathematical sciences dominate those that divide them.



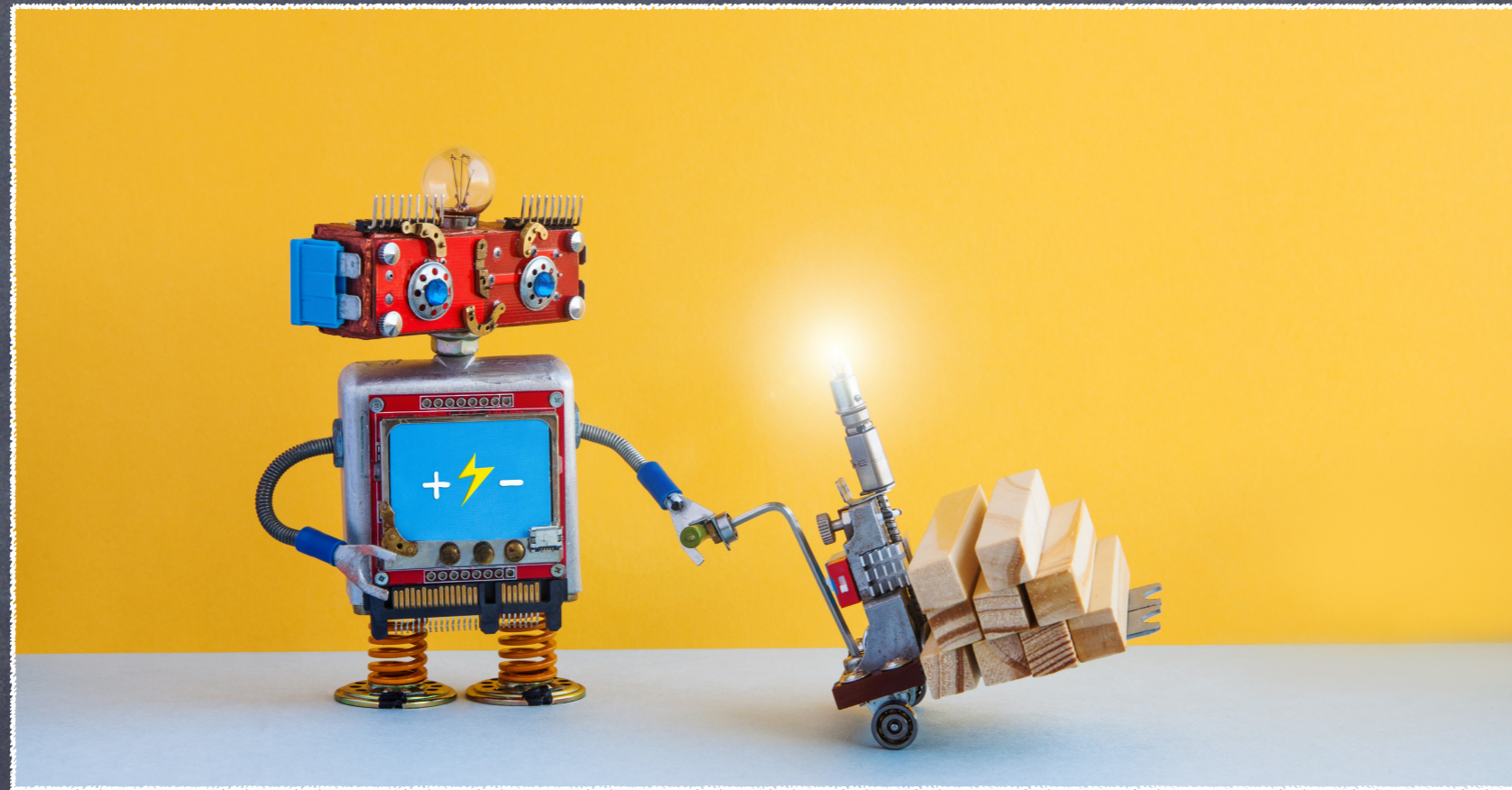
Why knowledge for the pure sake of
knowing is good enough to justify
scientific research

Sheila Patek, Duke University

Duke University biologist Sheila Patek has faced criticism from lawmakers over her research into mantis shrimp and trap-jaw ants, with some calling her government-funded studies a waste of taxpayer money.

But according to Patek, not only do her findings have important practical applications, but scientific inquiry is most fruitful when knowledge is sought for its own sake, not to justify budgets.

PBS News Hour Commentary



Computer assisted proofs

- Formal verification is a rapidly growing field of computer assisted proof verification.
- Formal verification offers an assurance of correctness far beyond other forms of testing.
- In safety critical systems, even an occasional failure is not acceptable.
- The driverless trains of Paris metro use formal systems for verifications of their algorithms.

- As formal verification grows in power, one could hope to use it many other areas and to a much greater extent.
- Yet - this is a small field with a small community working in it so that the potential for making significant contribution is high.

```
(function (ko, datacontext) {  
  <div style="background-image: url('background-image.png'); background-color: yellowgreen; padding: 10px; text-align: center; color: white; font-weight: bold; font-size: 1.2em;">The image can be tiled across the background, while the text runs across the top.  
  // persisted properties  
  <html> <p style="font-weight: bold;">HTML font code is done using the <code> tag.  
  <html> <body style="background-color: yellowgreen; color: white; padding: 10px; text-align: center; font-weight: bold; font-size: 1.2em;">  
  <html> text - :200px; <code> <code> persisted properties  
  // Non - text - :200px; <code> persisted properties  
  <html> <errorMessage = ko , observable() ; ko , observable() ;  
  <p style="color: orange;">HTML font code is done using the <code> tag.  
  function todoitem(data) ;  
  var self = this  
  data = data || {}  
  // Non - persisted properties  
  <html> <errorMessage = text - :200px; ko , observable() ; ko , observable() ;  
  <p style="font-weight: bold;">HTML font code is done using the <code> tag.  
  <body style="background-color: yellowgreen; color: white; padding: 10px; text-align: center; font-weight: bold; font-size: 1.2em;">  
  text - :200px; <code> <code> persisted properties  
  - text - :200px; <code> persisted properties  
  <errorMessage = ko , observable() ; ko , observable() ;  
  style="color: orange;">HTML font code is done using the <code> tag.  
}
```

Can we keep up with AI?

A tank that drives itself. A drone that picks its own targets. A machine gun with facial-recognition software.

Sound like science fiction? Weapons powered by artificial intelligence are already here.

Jonah M. Kessel/The New York Times

Killer Robots Aren't Regulated. Yet.

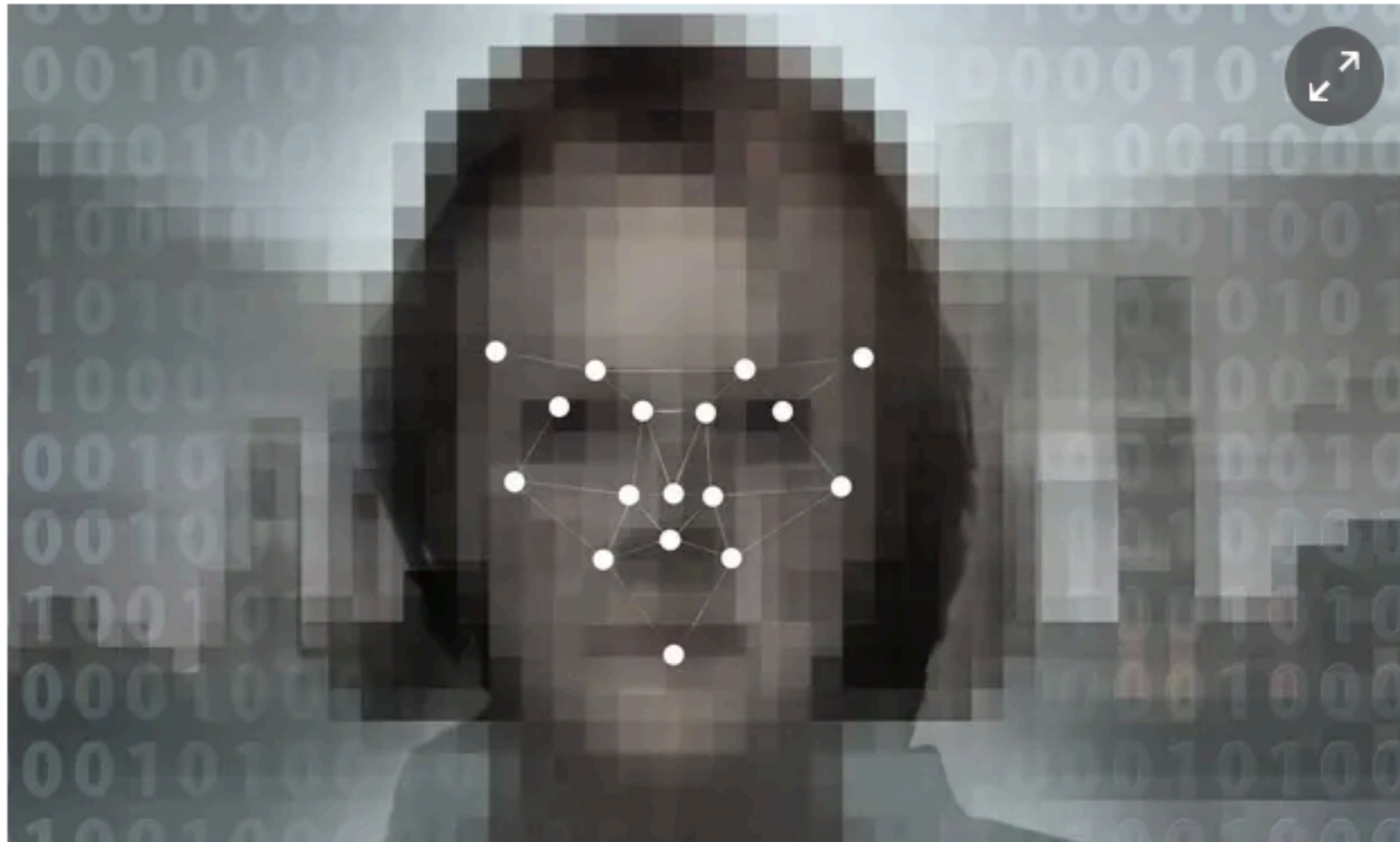
"Killing in the Age of Algorithms" is a New York Times documentary examining the future of artificial intelligence and warfare.



Jonah M. Kessel/The New York Times

AI expert calls for end to UK use of 'racially biased' algorithms

Prof Noel Sharkey says systems so infected with biases they cannot be trusted



▲ Facial recognition technology has also come under scrutiny. Photograph: Fanatic Studio/Gary Waters/Getty/Collection Mix: Subjects RF

TOM SIMONITE

BUSINESS 12.03.2019 07:00 AM

Senators Protest a Health Algorithm Biased Against Black People

A study found that the formula discriminated against black people by counting health care costs as an indicator of illness.



Recruitment algorithms are ‘infected with biases’, AI expert warns

By **Ashleigh Webber** on 13 Dec 2019 in **Unconscious bias, Artificial intelligence, Equality & diversity, Latest News, Recruitment & retention, Pre-employment screening**



The use of algorithms in “life-changing” decisions, such as recruitment, should be stopped immediately as they are often “infected with biases”, according to an expert on artificial intelligence.

BLOG 05 SEPTEMBER 2019

Algorithm Bias in Credit Scoring: What's Inside the Black Box?

By [Maria Fernandez Vidal](#), [Jacob Menajovsky](#)

As digital financial services (DFS) expand around the world with the promise of improving financially excluded customers' access to more affordable products and services, the growing use of algorithms opens opportunities but also the possibility for unfair bias and discrimination.

The past few years have seen growing recognition that machine learning raises novel challenges for ensuring non-discrimination, due process, and understandability in decision-making.

In particular, policymakers, regulators, and advocates have expressed fears about the potentially discriminatory impact of machine learning, with many calling for further technical research into the dangers of inadvertently encoding bias into automated decisions.



Should Climate change
concern you?

No challenge poses a greater threat to our children, our planet, and future generations than climate change.

There is no doubt left about
causation economy - carbon
emission - global warming -
climate catastrophe.

The possibility of unexpected changes
is very much there.

The finiteness of the world did not
play a role in the past. There was
enough room for polluting it.

The Trouble with monsoon and the threatening problems with the great rivers caused by the melting of the Himalayan glaciers will cause misery sooner and more heavily in India than most other places.

The Trouble with monsoon and the threatening problems with the great rivers caused by the melting of the Himalayan glaciers will cause misery sooner and more heavily in India than most other places.

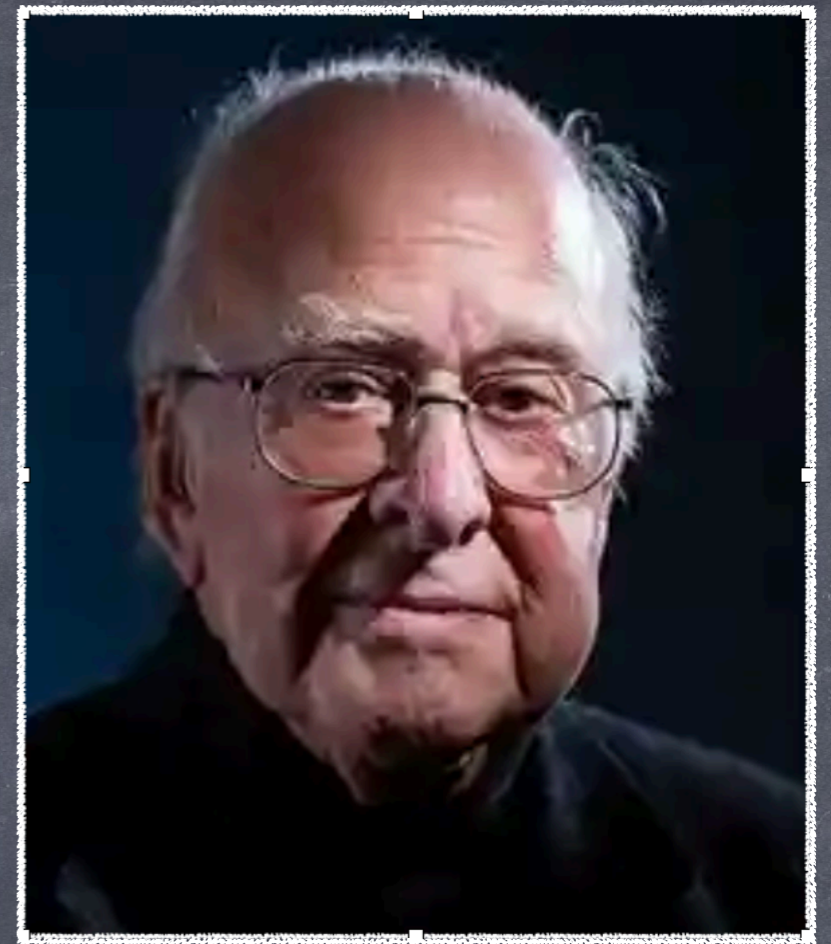
Technological research into efficient agricultural practices (and food production, in general) must be continued vigorously.

Act III

Call to Action

Basic science research deserves support, because in history, it was always true that research pursued for its own sake was what led to huge, generally unexpected, technological consequences that made life in the world easier.

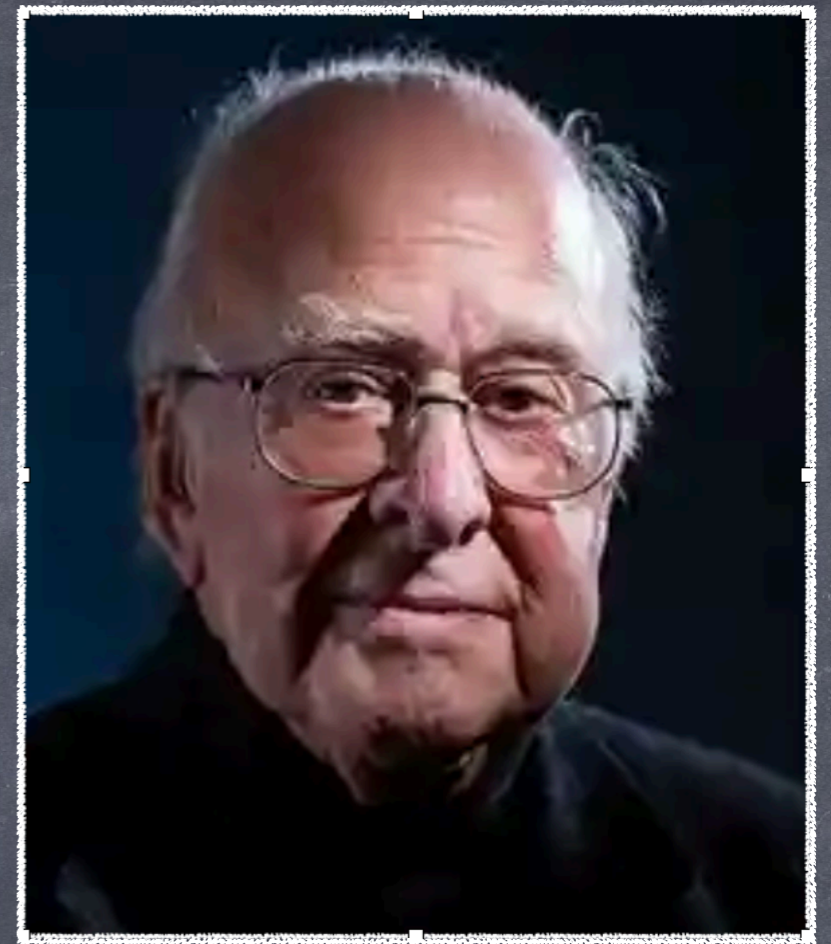
But it should also be emphasised that basic science cannot work on order.



Peter Higgs, the British physicist who gave his name to the Higgs boson, believes no university would employ him in today's academic system because he would not be considered "productive" enough.

The Guardian, Decca Aitkenhead Friday, Dec 6, 2013

Physicist doubts work
like Higgs boson
identification achievable
now as academics are
expected to 'keep
churning out papers'



Peter Higgs, the British physicist who gave his name to the Higgs boson, believes no university would employ him in today's academic system because he would not be considered "productive" enough.

The Guardian, Decca Aitkenhead Friday, Dec 6, 2013

Continuous Evaluation
of research is no good

Continuous Evaluation Of research is no good

- Evaluation of research output at short intervals is doing more harm than good.

Continuous Evaluation Of research is no good

- Evaluation of research output at short intervals is doing more harm than good.
- While the volume of our research output has increased over the years, the depth is certainly lacking.

Some thoughts for the future

Some thoughts for the future

- Evaluation of research should be based on the quality attested by experts in the field in the form of reports. Sharing them would do a lot of good.

Some thoughts for the future

- Evaluation of research should be based on the quality attested by experts in the field in the form of reports. Sharing them would do a lot of good.
- For instance, mandate that detailed reviews of research projects be sought for making a funding decision and then share them with the concerned researcher.

Recognising the value of
teaching

Recognising the value of teaching

- The effort necessary to impart high quality teaching hasn't been recognised.

Recognising the value of teaching

- The effort necessary to impart high quality teaching hasn't been recognised.
- The impact of lively and well prepared lectures is usually ignored.

Recognising the value of teaching

- The effort necessary to impart high quality teaching hasn't been recognised.
- The impact of lively and well prepared lectures is usually ignored.
- On the ground that teaching evaluation is subjective, it is mostly ignored.

Thank you